In keeping with the approximation involved in the calculation of the inviscid flowfield, the term $\bar{\tau}_\infty$ has been neglected.

Apparent Body Surface Displacement

The displacement of the inviscid dividing streamline ($\psi = 0$) which will align streamlines in the inviscid flow with corresponding streamlines outside the boundary layer in the actual flow is designated here to be $\delta^{**}(x)$. This displacement can be related to the displacement thickness [$\delta^*(x)$, defined by Eq. (18)] as is shown in the following.

Consider some particular streamline ψ^+ outside the boundary layer. Its location may be computed from

$$y^+ = \frac{1}{r^\epsilon} \int_{\psi_w}^{\psi^+} \frac{d\psi}{\bar{\rho}\bar{u}} \qquad (20)$$

Similarly, for the inviscid flow

$$y^+ - \delta^{**} = \frac{1}{r^\epsilon} \int_0^{\psi^+} \frac{d\psi}{\rho_i u_i} \qquad (21)$$

Also, since for $\psi < 0$, $\rho_i u_i = \rho_e u_e$

$$0 = \frac{1}{r^\epsilon} \int_{\psi_w}^0 \frac{d\psi}{\rho_i u_i} + \frac{\psi_w}{\rho_e u_e r^\epsilon} \qquad (22)$$

Equations (21) and (22) are subtracted from Eq. (20) giving

$$\delta^{**} = \frac{1}{r^\epsilon} \int_{\psi_w}^{\psi^+} \left[\frac{1}{\bar{\rho}\bar{u}} - \frac{1}{\rho_i u_i} \right] d\psi - \frac{\psi_w}{\rho_e u_e r^\epsilon} \qquad (23)$$

The integral may now be transformed using $d\psi = \bar{\rho}\bar{u}\, r^\epsilon dy$, so that

$$\delta^{**} = \delta^* - (\psi_w / \rho_e u_e r^\epsilon) \qquad (24)$$

This is the usual result except for the more general definition of δ^*.

Conclusions

The integral momentum equation has been generalized to include flows with entropy gradients across inviscid streamlines. A related equation for total enthalpy of the mean flow ($H = h + u^2/2$) can also be obtained. However, since its development is unaffected by such entropy gradients, it is not discussed herein. The generalized integral momentum equation together with the integral total enthalpy equation should be useful in calculating approximate boundary-layer solutions in flows which have passed through curved shocks.

Reference

[1]Schubauer, G. B. and Tchen, C. M., "Turbulent Flow," *Turbulent Flows and Heat Transfer, Volume V, High Speed Aerodynamics and Jet Propulsion,* Princeton University Press, Princeton, N.J., 1959, pp. 75-195.

Noise Produced by Fluid Inhomogeneities

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I. Introduction

THERE is currently in progress a substantial amount of research concerning the noise production of hot spots passing through extreme velocity gradients. This process is

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found in turbopropulsion systems where hot spots produced in the combustor pass through nozzles in the turbine assembly. The mechanism of sound production by this process has been known for some time, was explicitly considered in numerous investigations of entropy wave instability in rocket engines and motors, and was quantitatively presented in the calculations of Crocco and Sirignano.[1] The mechanism was considered by Cuadra,[2] and, more recently, was investigated concerning aeroacoustics implications by Candel.[3] Research along these lines has continued through the works of Zukoski[4] and Cumpsty,[5] and an application of the theory to core engine noise from an engine has been made by Pickett.[6] This kind of noise has been called entropy noise by the author[7] to distinguish it from noise directly generated by the turbulent combustion process, which may form another significant component of core engine noise.

Entropy noise from hot spots is a degenerate form of noise that may be caused by various kinds of fluid inhomogeneities. There are at least two basic mechanisms that will cause noise when a fluid element of different thermodynamic properties from the surrounding fluid attempts to traverse a given pressure (or velocity) gradient. The first is that the acceleration of the element in question may be different from the surrounding fluid by virtue of a different density. This is seen easily by examination of the momentum equation $Du/Dt = -\nabla p/\rho$, where D/Dt is the material time derivative u is the velocity vector and $\nabla p/\rho$ is the pressure gradient term. Clearly, ρ, the density, may be affected by temperature or molecular weight at a given value of the pressure. Inhomogeneities in the temperature and the molecular weight will give rise to different accelerations of neighboring elements, and an unsteady motion must be set up to alter ∇p. This alteration, of course, causes sound to be generated. A second mechanism of noise generation, and one considered explicitly in this paper, has not appeared to have been explored, however; this is caused by inhomogeneities in specific heat. For isentropic motion of perfect gases $(D\rho/Dt)(1/\rho) = -(Dp/Dt)/(\gamma p)$. Consequently, for a fixed pressure change, the fractional change in density depends upon γ, the ratio of specific heats. Inhomogeneities in this quantity would give rise to separation of fluid elements, which cannot be allowed by continuity considerations. Again, an unsteady adjustment must take place which is perceived as sound.

In an actual engine system, one mechanism for production of hot spots is the burning of various fluid elements at various different mixture ratios. Since a gas turbine combustor burns by a diffusion flame, whereby the air and fuel must first mix before they burn, there is no guarantee that an absolutely uniform air/fuel ratio can be maintained for every fluid element. Variable mixture ratio means, of course, variable temperature, and this is the effect that has been studied insofar as a noise source is concerned. However, variable mixture ratio also implies variable molecular weight and heat capacity. The purpose of this Note is to investigate whether either of these last two variations may be responsible for a significant noise source.

II. Analysis

The analysis will be done within the context of one-dimensional unsteady flow, as in the work of Candel.[3] A mixture of thermally perfect gases will be assumed as the working fluid. The fluid composition will be considered to consist of species 1 (which may also be a mixture) and a small and variable mole fraction of species 2, which has different molecular weight and specific heat as compared with species 1. Axial diffusion of species is assumed to be negligible, so that the space-wise variation of composition is convected by the flow velocity. Furthermore, small density variations are allowed due to temperature and molecular weight variations; there is also no axial diffusion of heat, and so the hot spots (or cold spots) are convected by the flow. One considers,

therefore, a mean flow of species 1 through a variable area channel, upon which there are superimposed small temperature and composition variations.

The linearized equations governing the disturbances are precisely those of Candel[3]:

$$\frac{\partial R}{\partial t} + \bar{u}\left(\frac{\partial R}{\partial z} + \frac{\partial U}{\partial z}\right) = 0 \tag{1}$$

$$\frac{\partial U}{\partial t} + \frac{d\bar{u}}{dz}(2U + R - P) + \bar{u}\frac{\partial U}{\partial z} + \frac{\bar{p}}{\bar{\rho}\bar{u}}\frac{\partial P}{\partial z} = 0 \tag{2}$$

$$\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial z}\right)S = 0 \tag{3}$$

where

$$R = \rho'/\bar{\rho} \quad U = u'/\bar{u} \quad P = p'/\bar{p} \quad S = s'/c_{v_1}$$

In the preceding, ρ is density, u is velocity, p is pressure, s is entropy per mole, c_{v_1} is the molar heat capacity at constant volume (assumed constant) for species 1, z is the axial direction, t is time, barred quantities are the steady-state quantities (z-dependent), and primed quantities are the small deviations of ρ, u, p, and s from the steady-state quantities. Equation (3) contains the no-diffusion assumption that all entropy disturbances are only convected by the mean flow.

For a mixture of thermally perfect gases

$$P = R + \Gamma - W \tag{4}$$

where $\Gamma = T'/\bar{T}$ and $W = M'/\bar{M}$ with T the temperature and M the molecular weight. Let $\mathcal{E}$ be the small mole fraction of species 2, for which the no-diffusion assumption assures

$$\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial z}\right)(\mathcal{E} \text{ or } W) = 0 \tag{5}$$

Then the entropy for thermally and calorically perfect gases is given by

$$s = (1 - \mathcal{E})[c_{p_1}\ln T/T_r - R_0 \ln p/p_r - R_0\ln(1 - \mathcal{E}) + s_{r_1}]$$
$$+ [c_{p_2}\ln T/T_r - R_0\ln p/p_r - R_0\ln\mathcal{E} + s_{r_2}] \tag{6}$$

where c_p is the molar heat capacity at constant pressure and T_r, p_r, and s_r are reference values of temperature, pressure, and entropy. Here R_0 is the universal gas constant. Retaining terms of the order of primed quantities, $\mathcal{E}$, and $\mathcal{E}\ln\mathcal{E}$ in Eq. (6), there results

$$S = (\mathcal{E}/c_{v_1})[s_{r_2} - s_{r_1} + R_0(1 - \ln\mathcal{E})$$
$$+ (c_{p_2} - c_{p_1})\ln\bar{T}/T_r] + P - \bar{\gamma}R + \bar{\gamma}W \tag{7}$$

By use of the information from Eq. (5), and the fact that $T \propto (\bar{\rho})^{(\bar{\gamma}-1)}$ for isentropic flow, Eqs. (3) and (7) yield

$$\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial z}\right)\left[\frac{\mathcal{E}(c_{p2} - c_{p1})(\bar{\gamma} - 1)}{c_{v_1}}\ln\bar{\rho} + P - \bar{\gamma}R\right] = 0 \tag{8}$$

It may be verified readily that the first term of Eq. (8) reduces to a term proportional to γ', where γ is the ratio of specific heats. Equation (8) becomes

$$[\partial/\partial t + \bar{u}(\partial/\partial z)](P - \bar{\gamma}R - \bar{\gamma} G\ln\bar{\rho}) = 0 \tag{9}$$

with $G = \gamma'/\bar{\gamma}$. Letting σ be equal to $P - \bar{\gamma}R$, Eqs. (1, 2, and 9) differ from the Candel equations by the last term in Eq. (9).

Taking the Fourier transform of Eqs. (1, 2, and 9),

$$i\omega P_\omega + \bar{u}\frac{dP_\omega}{dz} + \bar{\gamma}\bar{u}\frac{dU_\omega}{dz} = \bar{\gamma}\bar{u}G_\omega\frac{d}{dz}\ln\bar{\rho} \tag{10}$$

$$i\omega U_\omega + \frac{d\bar{u}}{dz}\left[2U_\omega + P_\omega\left(\frac{1-\bar{\gamma}}{\bar{\gamma}}\right)\right] + \bar{u}\frac{dU_\omega}{dz} + \frac{\bar{p}}{\bar{\rho}\bar{u}}\frac{dP_\omega}{dz} = \frac{\sigma_\omega}{\bar{\gamma}}\frac{d\bar{u}}{dz} \tag{11}$$

$$i\omega\,\sigma_\omega + \bar{u}\frac{d\sigma_\omega}{dz} = \bar{\gamma}G_\omega\bar{u}\frac{d}{dz}\ln\bar{\rho} \tag{12}$$

where the subscript ω denotes the transformed quantity (or the amplitude of the quantity in the case of simple harmonic time dependence). Equation (12) may be solved directly to yield

$$\sigma_\omega = \sigma_{\omega 0}\exp -i\omega\int_0^z \frac{dz'}{\bar{u}} + \bar{\gamma}\exp -i\omega\int_0^z \frac{dz'}{\bar{u}}$$
$$\times \int_0^z G_\omega\bar{u}\frac{d}{dz'}(\ln\bar{\rho})\exp -i\omega\int_0^{z'}\frac{dz''}{\bar{u}}\,dz' \tag{13}$$

where $\sigma_{\omega 0}$ is the transform of σ at $z = 0$. Since

$$G = G(t - \int_0^z\frac{dz'}{\bar{u}}) \qquad G_\omega = G_{\omega 0}\exp -i\omega\int_0^z\frac{dz'}{\bar{u}}$$

so that Eqs. (10, 11, and 13) may be combined to yield

$$i\omega\,P_\omega + \bar{u}\frac{dP_\omega}{dz} + \bar{\gamma}\bar{u}\frac{dU_\omega}{dz} = \bar{\gamma}\bar{u}\frac{d}{dz}\ln\bar{\rho}G_{\omega 0}\times\exp -i\omega\int_0^z\frac{dz'}{\bar{u}} \tag{14}$$

$$i\omega\,U_\omega + \frac{d\bar{u}}{dz}\left[2U_\omega + P_\omega\left(\frac{1-\bar{\gamma}}{\bar{\gamma}}\right)\right] + \bar{u}\frac{dU_\omega}{dz} + \frac{\bar{p}}{\bar{\rho}\bar{u}}\frac{dP_\omega}{dz}$$
$$= \left[\frac{d\bar{u}}{dz}\frac{\sigma_{\omega 0}}{\bar{\gamma}} + G_\omega\int_0^z\bar{u}\frac{d}{dz}(\ln\bar{\rho})dz'\right]\exp -i\omega\int_0^z\frac{dz'}{\bar{u}} \tag{15}$$

Equations (14) and (15) are the Candel equations with new inhomogeneities on the right-hand sides involving G. Since $\bar{\gamma}\bar{u}d(\ln\bar{\rho})/dz$ is of the same order of magnitude as $d\bar{u}/dz$, the inhomogeneities are all of the same order if G is of the same order as σ; $\sigma_{\omega 0}$ will be nonzero if there are temperature and/or molecular weight inhomogeneities in the fluid; G will be nonzero if there are heat capacity inhomogeneities. These irregularities will cause sound to be generated according to Eqs. (14) and (15), which are two equations in U_ω and P_ω, when the irregularities pass through regions where $d\bar{u}/dz$ and $d\bar{\rho}/dz$ are nonzero. That portion of sound, generated due to the terms involving G, is herein called "gamma-prime" noise.

III. Discussion

In the absence of γ variations Eqs. (14) and (15), together with Eq. (4), show that the entropy variations due to temperature and molecular weight variations are equivalent as a sound source. This fact is being exploited in some current experimental research at this laboratory. However, whenever G is nonzero Eqs. (14) and (15) become severely complicated. Although the molecular weight depends on the molecular mass, of course, and the heat capacity depends upon the molecular structure, generally molecular weight variations will carry with them γ variations, even if the different species have the same structure. For example, even if species 2 were a heavy diatomic gas and species 1 were a light diatomic gas, the molecular weight variations introduced by a small $\mathcal{E}$ may still induce γ variations if the internal degrees of freedom were excited to different extents in the two different species. On the other hand, since γ is a ratio of c_p and c_v, and a fixed change

in c_p produces the same absolute change in c_v (for perfect gases), variations in γ tend to be suppressed.

In turbopropulsion systems, the use of excess air in the combustor together with preponderance of relatively inert N_2 makes γ' at least an order of magnitude lower than $T'/\bar{T}$ when caused by local mixture of ratio fluctuations. Furthermore $W'/\bar{W}$ also is negligible compared with $T'/\bar{T}$. In such cases, the complication of γ' noise does not need to be considered. In rocket engines, however, there may be some merit in investigating γ' noise as a source of chamber pressure roughness. As examples, consider some thermochemical computations of Ref. 8. For ethyl alcohol/liquid oxygen operating at a pressure ratio of 13.6 and a chamber pressure of 200 psia, the maximum specific impulse occurs at an oxidizer to fuel mass flow ratio of 1.4. Considering a variability of mixture ratio from 1.3 to 1.5, the temperature varies by about 6.5%, the molecular weight varies by 5.4%, and γ varies by 0.59%. However, both Γ and W are positive with an increase in mixture ratio, so that they tend to cancel one another in Eq. (4). Consequently, although it would take detailed numerical computations to confirm it, the γ' contributions can be of the same order as Γ and W contributions to noise generation. On the other hand, there are some propellant combinations, for example, liquid oxygen and gasoline, which have only weak γ variations in the vicinity of the I_{s_p} maximum. Finally, γ' noise also may arise in some industrial processes involving turbulent flow and mixing of dissimilar gases.

References

[1] Crocco, L. and Sirignano, W. A., "Behavior of Supercritical Nozzles under Three-Dimensional Oscillatory Conditions," AGARD ograph No. 117, NATO, 1967.

[2] Cuadra, C., "Acoustic Wave Generation by Discontinuities Flowing Past an Area Change," Journal of the Acoustical Society of America, Vol. 42, Oct. 1967, pp. 725-732.

[3] Candel, S. M., "Analytical Studies of Some Acoustic Problems of Jet Engines," Ph.D. thesis, California Institute of Technology, 1972.

[4] Zukoski, E., "Temperature Distortion Effects on Turbine and Afterburner Noise," American Society of Mechanical Engineers, Paper 75-GT-40, 1975.

[5] Cumpsty, N. A., "Excess Noise from Gas Turbine Exhausts," American Society of Mechanical Engineer, Paper 75-GT-61, 1975.

[6] Pickett, G. F., "Core Engine Noise Due to Temperature Fluctuations Convecting Through Turbine Blade Rows," AIAA Paper 75-528, 1975.

[7] Strahle, W. C., "A Review of Combustion Generated Noise," Aeroacoustics: Jet and Combustion Noise; Duct Acoustics, edited by Nagamatsu, AIAA, New York, 1975, pp. 229-248.

[8] Sutton, G. P., Rocket Propulsion Elements, Wiley, New York, 1956, pp. 114-121.

Technique for Atmospheric Rate Chemistry Calculations

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Introduction

THE possibility that predictions of atmospheric photo-chemistry/transport models are sensitive to uncertainties in reaction rates and other inputs underscores the need for rapid numerical integration schemes in rate photochemistry

problems.[1,2] Reducing the computational burden has an obvious advantage in facilitating sensitivity studies to assess the influence of such uncertainties on, say, predicted ozone diminutions from nitrogen oxides ($NO_x = NO + NO_2$) in the exhaust plume of SST engines.

Analysis and Results

An algorithmic approach to integration of rate chemistry problems in combustion has been developed by Rubel and Baronti and described by Ferri.[3,4] In this approach, the production rate for the "ith" chemical species is written:

$$dX_i/dt = -A_i X_i + B_i \tag{1}$$

where X_i, A_i, and B_i are, respectively, the concentration, loss coefficient, and gain coefficient. Integration of Eq. (1) yields

$$X_{i_t} = \exp\left(-\int_0^t A_i dt\right)\left[X_{i_0} + \int_0^t B_i \exp\left(\int_0^t A_i dt\right) dt\right] \tag{2}$$

where X_{i_0} and X_{i_t} are the concentrations at the beginning and end of the time step, respectively. According to Rubel and Baronti,[5] if the time step is small enough and the temperature is fixed during the time step, Eq. (2) can be approximated

$$X_{i_t} \approx X_{i_0} \exp\left(-A_{i_0}\Delta t\right) + \frac{B_{i_0}}{A_{i_0}}\left[1 - \exp\left(-A_{i_0}\Delta t\right)\right] \tag{3}$$

where Δt is the duration of the time step, and A_{i_0} and B_{i_0} are loss and gain coefficients at the start of the time step. The validity of the approximation can be determined by observing the effect of shortened Δt upon results obtained using Eq. (3).

In a recent consideration of the plume chemistry for a current SST engine, the GE-4 engine, Gupta and Grose utilized a 20-reaction chemical model.[6] They did note however, that a simplified five-reaction model is accurate to 10%. These five reactions are listed in Table 1, along with species production-rate equations, reaction rates, and photodissociation coefficients. The photodissociation coefficients are those of Gupta and Grose.[6] Reaction rates have been calculated from the tabulation of Wuebbles and Chang[7] and Stewart and Hoffert,[8] for a temperature of 220 K and a height of 20 km. Reaction rate k_1, when corrected for the number density at 20 km ($M = 2 \times 10^{18}$ cm^{-3}) and converted to units of ppm^{-1} sec.$^{-1}$, is 12% higher than the rate utilized by Gupta and Grose.[6]

The set of reactions listed in Table 1 have been solved using a pocket electronic calculator for the following initial conditions. Volumetric molecular oxygen concentration is constant at 0.21 and total particle number density M is constant an 2×10^{18} cm^{-3}. The initial ozone concentration is 1.5×10^{-6}.

We have considered two projected SST engines. One of these is the GE-4 considered by Gupta and Grose.[6] The other is based on experiments discussed by Ferri[9] regarding NO_x effluent reductions that are within the reach of current technology. For the GE-4 engine,[6] initial concentrations of NO and NO_2 are, respectively, 5×10^{-7} and 5×10^{-9}. For the SST improvement projected by Ferri, initial NO and NO_2 concentrations are, respectively, 10^{-7} and 10^{-9}.

Because the ambient atmospheric concentration of $O(^3P)$ is so low,[10] we have set it at zero. We also have assumed that the function $R = 1$, where R is defined as[6]

$$R = \frac{\partial O/\partial t \text{ Depletion}}{\partial O/\partial t \text{ Photoproduction}} \tag{4}$$

If R were greater than 1, the small amount of atomic oxygen in the plume soon would be depleted completely. On the other hand, if $R < 1$, ozone depletion rates will be lower than calculated. By maintaining $R = 1$, we are overestimating ozone depletion.